Fourier transform of chirped pulses

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We calculate the Fourier transform of chirped pulses for a few pulse shapes. In the present context, we define chirped pulses by a linear dependence of the instantaneous frequency on time, i.e. $\omega(t) = \omega_0 + ct$. Another option would be to define it as a quadratic spectral phase, which is equivalent for Gaussian pulses. The pulses are assumed to be peaked around $t = 0$, as a shift in time is trivially achieved by multiplying the Fourier transform by $\exp(i\omega t_0)$. All pulses are assumed to be expressed in the form

$$E(t) = f(t)\cos(\phi_0 + \omega_0 t + ct^2) \tag{1}$$
$$= \frac{1}{2} f(t) e^{i(\phi_0+\omega_0 t+ct^2)} + c.c. \tag{2}$$
$$= \frac{1}{2} g(t) e^{i(\phi_0+\omega_o t)} + c.c.\,, \tag{3}$$

where $f(t)$ is the (real) envelope function, while $g(t) = f(t)\exp(ict^2)$ is in general complex. ϕ_0 is the carrier-envelope phase. Here and in the following we assume that $f(0) = 1$ and neglect the trivial scaling with peak field strength. The Fourier transform is then given by

$$\tilde{E}(\omega) = \frac{e^{i\phi_0}}{2}\tilde{g}(\omega-\omega_0) + \frac{e^{-i\phi_0}}{2}\tilde{g}^*(\omega+\omega_0) \tag{4}$$

where

$$\tilde{h}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} h(t)e^{-i\omega t}\mathrm{d}t \tag{5}$$

I. GAUSSIAN PULSE

A Gaussian pulse is described by $f(t) = \exp\left(-\frac{t^2}{2\sigma^2}\right)$, where σ is the standard deviation. We can then write $g(t) = \exp(-zt^2)$, where $z = \frac{1}{2\sigma^2} - ic$. The Fourier transform is then

$$\tilde{g}(\omega) = \frac{1}{\sqrt{8z}} e^{-\omega^2/4z}\,. \tag{6}$$

II. $\cos^2$ PULSE

This pulse is defined by

$$f(t) = \begin{cases} \cos\left(\pi t/T\right)^2 & |t| < T/2 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

By expanding the trigonometric function in polynomials, we can write (for $|t| < T/2$):

$$g(t) = \frac{1}{2}e^{ict^2} + \frac{1}{4}e^{ict^2-2i\pi t/T} + \frac{1}{4}e^{ict^2+2i\pi t/T} \tag{8}$$

which can be Fourier-transformed by using

$$\frac{1}{\sqrt{2\pi}}\int_{-T/2}^{T/2} e^{iat+ibt^2}\mathrm{d}t = \frac{\exp(-\frac{ia^2}{4b})}{i\sqrt{8ib}}\left[\operatorname{erf}\left(\frac{a-bT}{\sqrt{4ib}}\right) - \operatorname{erf}\left(\frac{a+bT}{\sqrt{4ib}}\right)\right] \tag{9}$$

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