Units of Fourier-transformed fields in `laserfields`

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We discuss the units of the Fourier transform of the laser fields as calculated in the `laserfields` library and the included `plotlaserfourier` and `printlaserfourier` programs.

The instantaneous intensity of an electric field is $I(t) = \frac{\epsilon_0 c}{2} E^2(t)$, which gives $I(t) = 3.50944 \cdot 10^{16}\, E_{au}^2(t)$ W/m^2 if the electric field is given in atomic units, $E(t) = E_{au}(t)\mathcal{E}_{au}$ (where $\mathcal{E}_{au} = \frac{e}{4\pi\epsilon_0 a_0^2} = 5.142207 \cdot 10^{11}$ V/m is the atomic unit of electric field strength). In atomic units, we have $I(t) = 5.45249\, E_{au}^2(t)$ a.u., where "a.u." here is Hartree/$(t_{au} a_0^2)$, with Hartree = 27.211 eV, atomic unit of time $t_{au} = 24.188$ as, and Bohr radius $a_0 = 5.29177 \cdot 10^{-11}$ m.

The Fourier transform as calculated in `laserfields` is

$$\tilde{E}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\omega t} E(t) \mathrm{d}t\,, \tag{1}$$

with all quantities in atomic units. This means that $\tilde{E}(\omega)$ from `laserfields` is in atomic units, which are $\mathcal{E}_{au} t_{au} = 1.24384 \cdot 10^{-5}$ V s/m. According to the Plancherel theorem, we have

$$\int_{-\infty}^{\infty} E^2(t) \mathrm{d}t = \int_{-\infty}^{\infty} |\tilde{E}(\omega)|^2 \mathrm{d}\omega \tag{2}$$

where we have used that $E(t)$ is real, i.e., $|E(t)|^2 = E^2(t)$. We use this to define a "spectral intensity" $\tilde{I}(\omega)$

$$\rho_{tot} = \int_{-\infty}^{\infty} I(t) \mathrm{d}t = \frac{\epsilon_0 c}{2} \int_{-\infty}^{\infty} E^2(t) \mathrm{d}t = \frac{\epsilon_0 c}{2} \int_{-\infty}^{\infty} |\tilde{E}(\omega)|^2 \mathrm{d}\omega = \int_{-\infty}^{\infty} \tilde{I}(\omega) \mathrm{d}\omega \tag{3}$$

where ρ_{tot} is the total energy density (per area) of the pulse (units, e.g., J/cm^2). Therefore, the spectral intensity $\tilde{I}(\omega)$ is given by $\tilde{I}(\omega) = 2.05338 \cdot 10^{-17} |\tilde{E}_{au}(\omega)|^2$ J s/cm^2, where $\tilde{E}_{au}(\omega)$ is the output of, e.g., `plotlaserfourier` or `printlaserfourier`. In atomic units, the spectral intensity is $\tilde{I}(\omega) = 5.45249\, |E_{au}(\omega)|^2$ a.u., where "a.u." here is Hartree t_{au}/a_0^2. Note that the prefactors in $I(t)$ and $\tilde{I}(\omega)$ are the same in atomic units, as it is simply $\epsilon_0 c/2$ in atomic units $(4\pi\epsilon_0 = 1, c = 1/\alpha)$, with $\alpha \approx 1/137.036$ the fine structure constant.

Finally, note that the integral in frequency ω runs from $-\infty$ to ∞, while `plotlaserfourier` by default only outputs $\omega > 0$. To perform the full integral, you can use that $\tilde{E}(-\omega) = \tilde{E}^*(\omega)$ since $E(t)$ is real, so that $\tilde{I}(-\omega) = \tilde{I}(\omega)$ and $\int_{-\infty}^{\infty} \tilde{I}(\omega)\mathrm{d}\omega = 2\int_0^{\infty} \tilde{I}(\omega)\mathrm{d}\omega$. So it would be possible to add a factor 2 to $\tilde{I}(\omega)$ and have it defined only for $\omega > 0$.

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